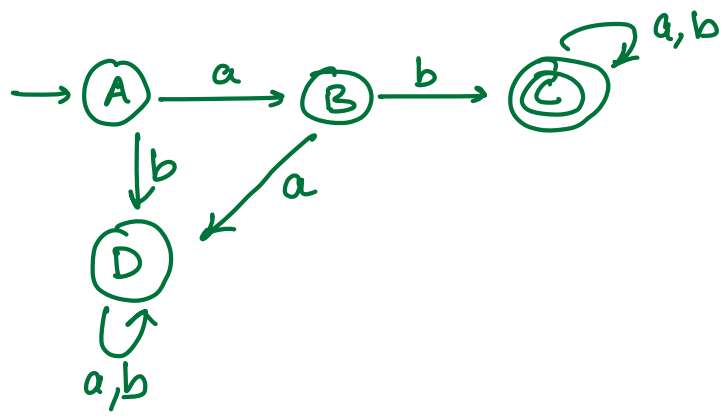
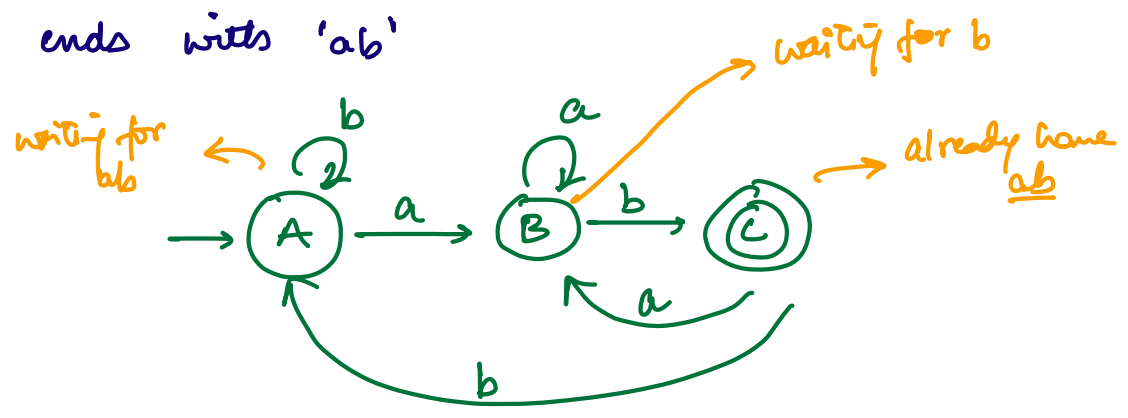


Q: Starts with 'ab'



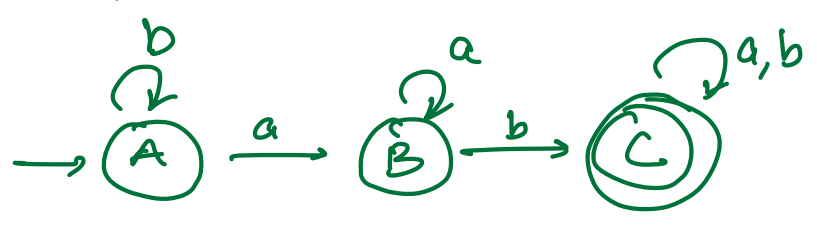
ab ✓
 ab~~a~~b ✓
 abb ✓
abab ✓
 a~~a~~b ✗
ba } ✗
 bb } ✗

Q: ends with 'ab'



✓ ab
 aab
 baab
 aabab
 abaaa
abbb
 A \xrightarrow{a} B \xrightarrow{b} C \xrightarrow{b} ?
 ●

Q: contains 'ab'?



baba

Operations

1. Union

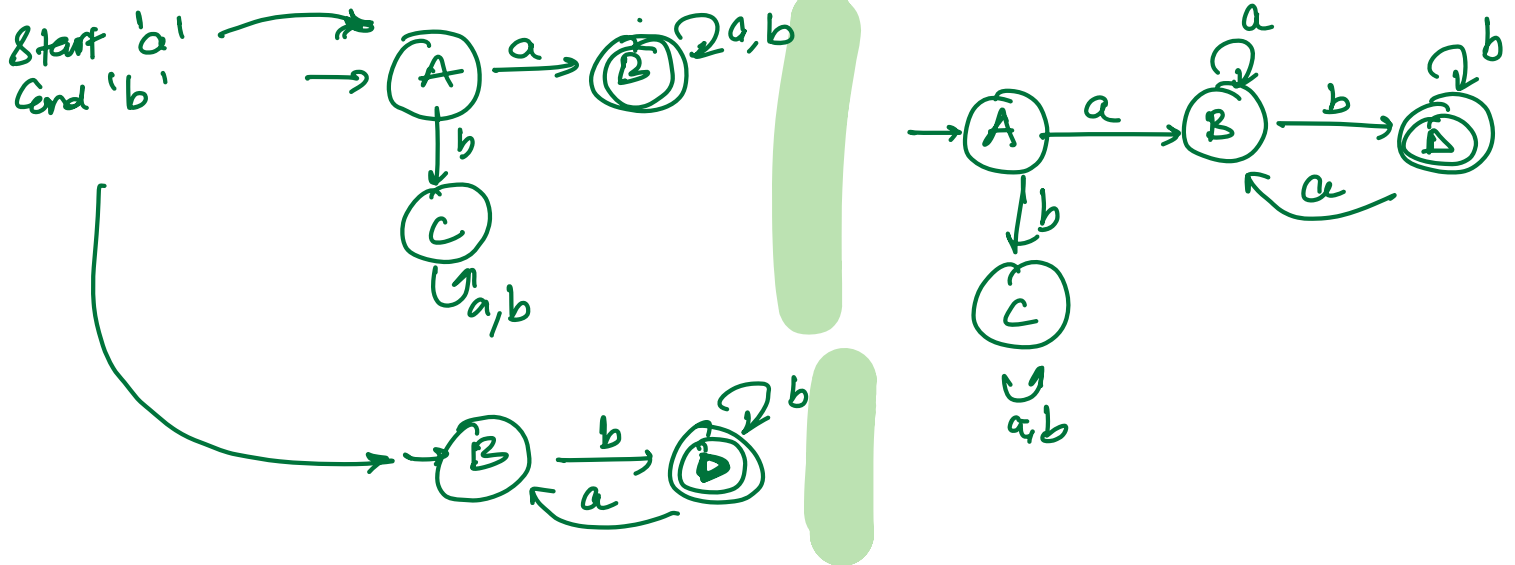
$\Sigma = \{a, b\}$

eg: L: starts & ends with different symbols

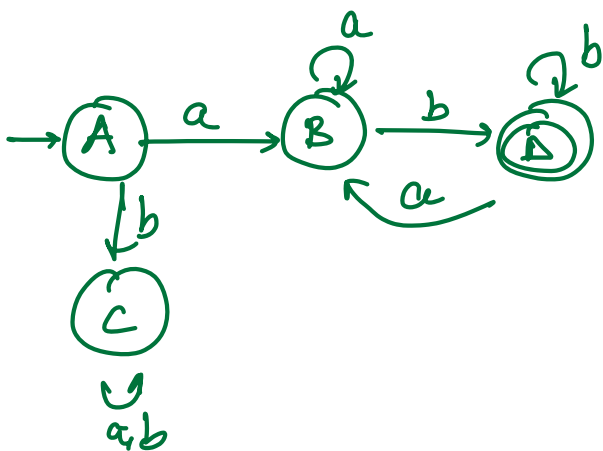
L_1 : starts with a & ends with b = $\{ab, abb, aab, \dots\}$

L_2 : starts with b & ends with a = $\{ba, bba, \dots\}$

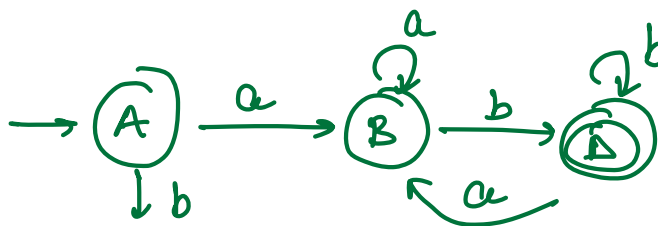
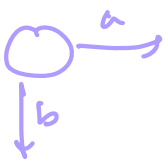
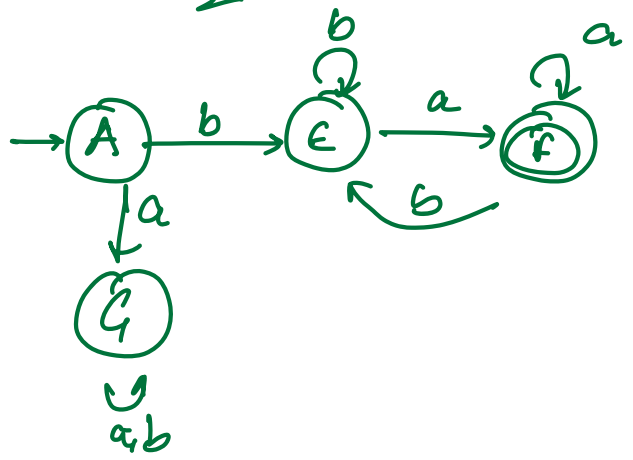
DFA for L_1 :



L_1 :



L_2



bab



2. Concatenation

DFA: starts with 'a' and ends with 'b'

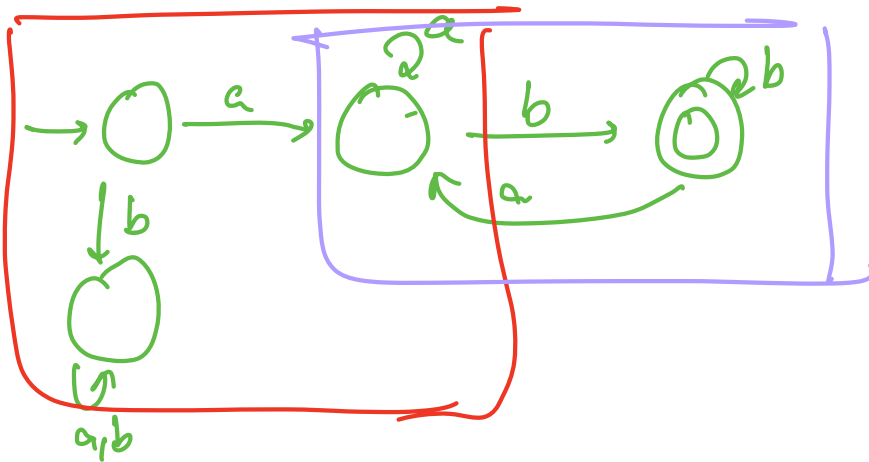
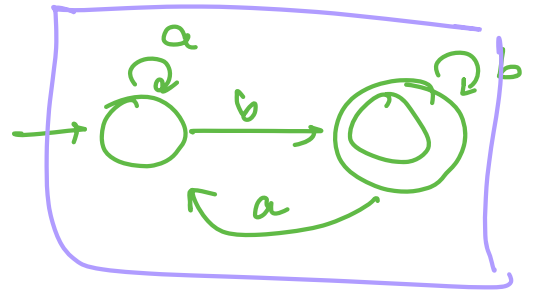
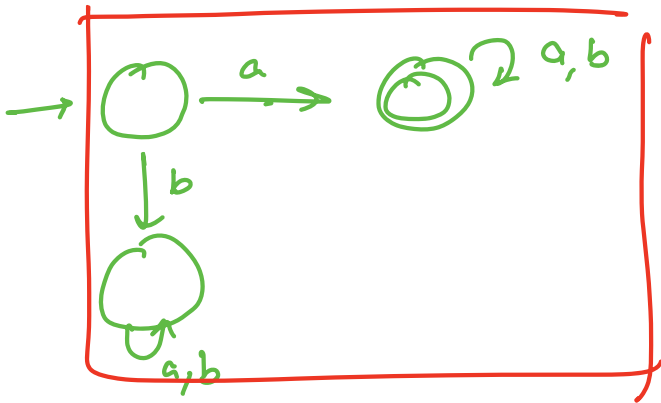
L_1 L_2

L_1 : starts with 'a'

$= \{a, aa, ab, aab, \dots\}$

L_2 : ends with b

$= \{b, bb, ab, b, \dots\}$



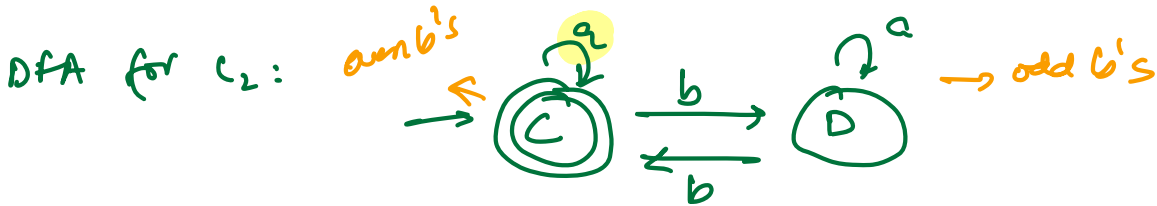
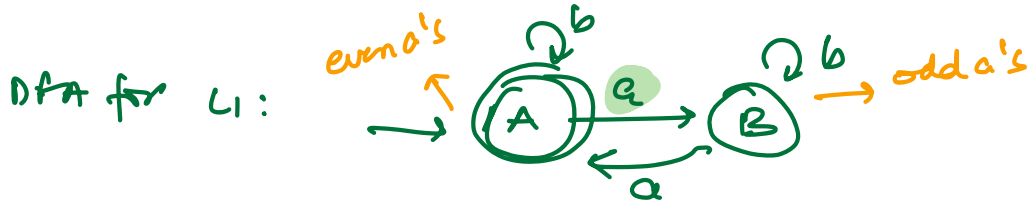
$$L_1 \rightarrow D_1$$

$$L_2 \rightarrow D_2$$

$$L_1.L_2 \rightarrow D_1.D_2$$

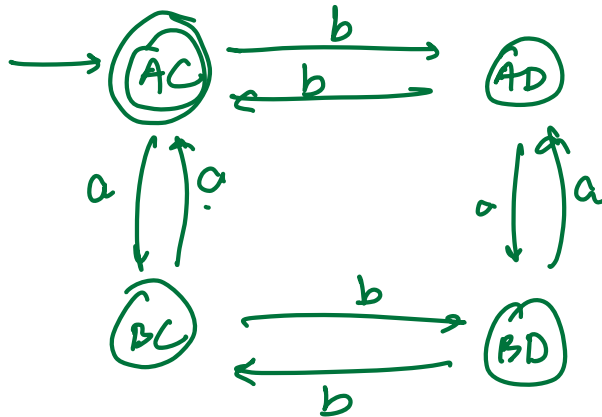
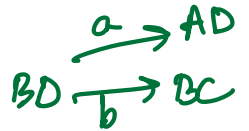
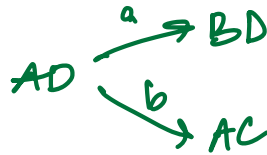
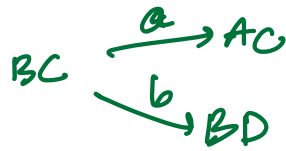
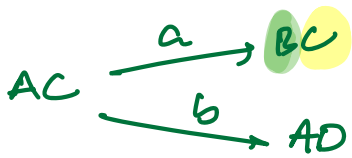
3. Cross Product

eg: Even no of a's and Even no of b's
 L_1 L_2



$$\{A, B\} \times \{C, D\}$$

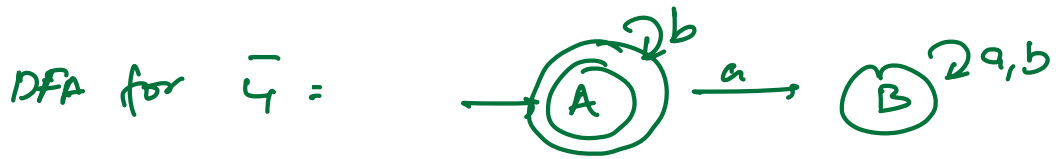
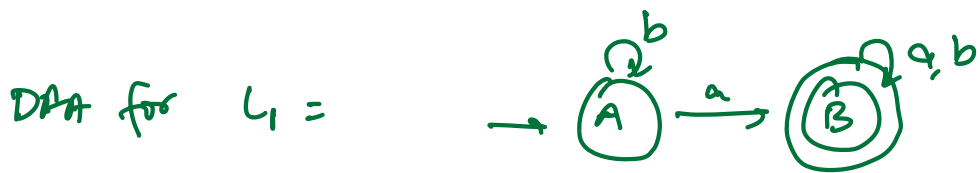
$$\{AC, BC, AD, BD\}$$



4. Complementations

eg: $L_1 = \text{contains 'a'} = \{a, aa, ba, aab, \dots\}$

$\bar{L}_1 = \text{not containing 'a'} = \{ \epsilon, b, bb, bbb, \dots \}$

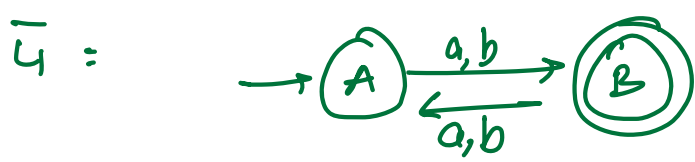
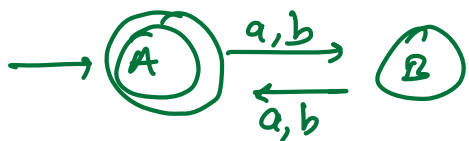


make final state \rightarrow non final state
 non final state \rightarrow final

$L_1 = \{ Q, \epsilon, \delta, q_0, f \}$

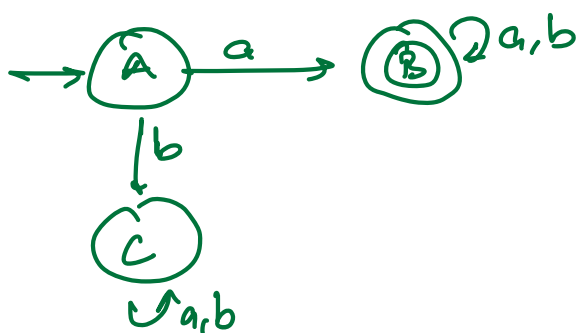
$\bar{L}_1 = \{ Q, \epsilon, \delta, q_0, Q-f \}$

Ex: $L_1 = \text{set of all even length strings}$

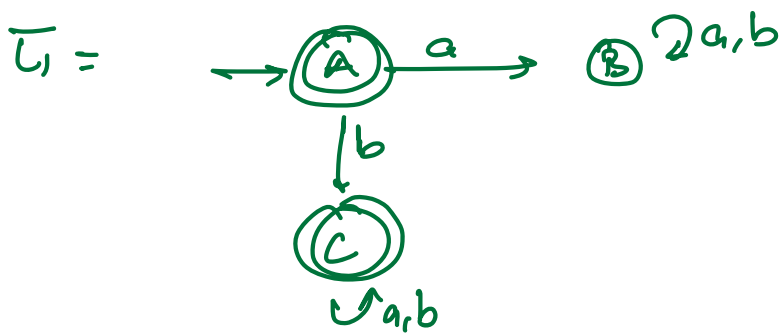


DFA for odd length string

Ex: $L_1 = \text{starts with 'a'}$



$Q = \{ A, B, C \}$
 $F = \{ B \}$



$F = \{A, C\}$

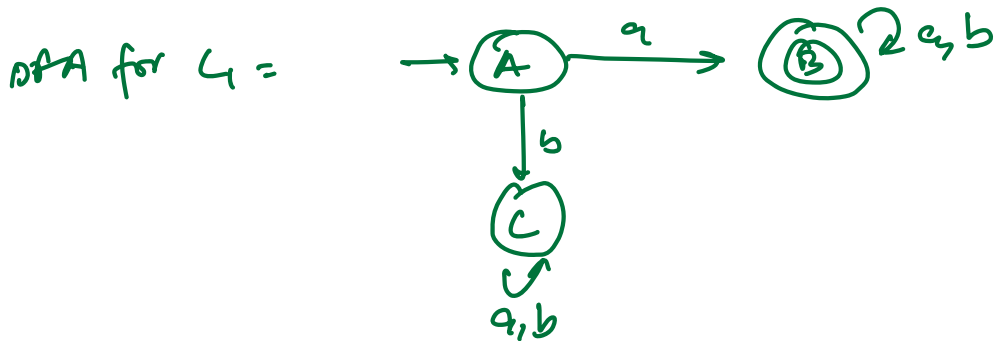
5. Reversal

$L_1 =$ starts with 'a'

$= \{a, aa, aab, ab, \dots\}$

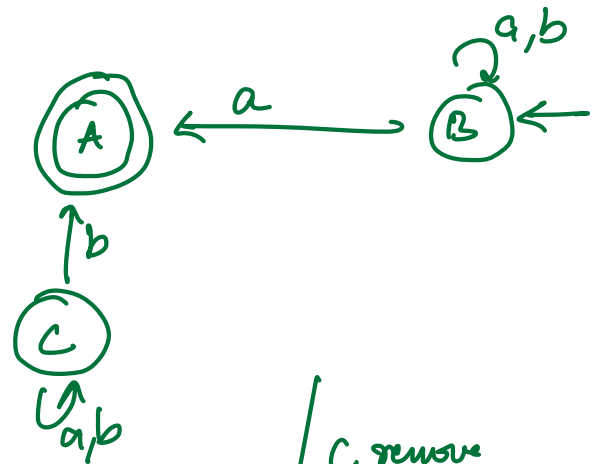
$L_1^R =$ take each string of L_1 and reverse it

$= \{a, aa, ba, ba, \dots\}$

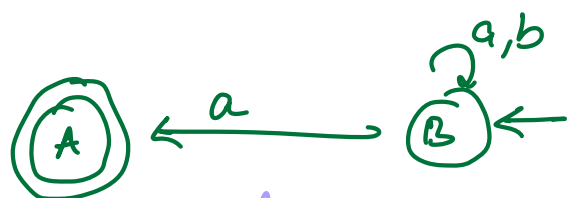


DFA for L_1^R :

- draw the states as it is
- final state \rightarrow initial state
- initial state \rightarrow final state
- reverse the edges



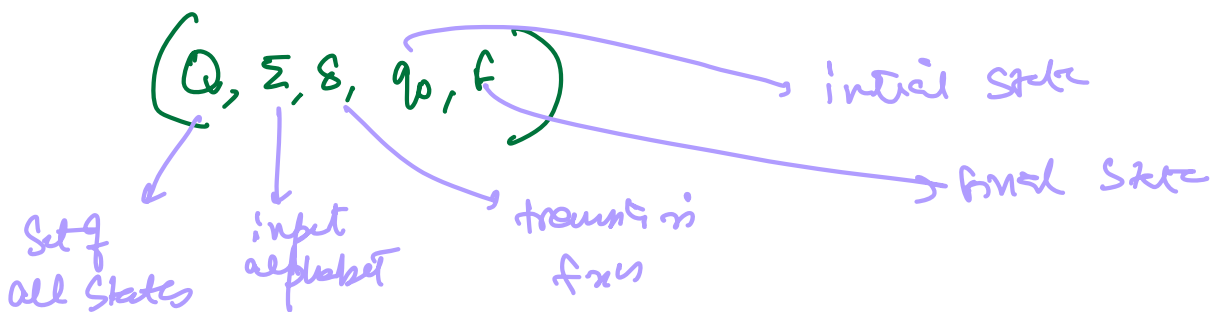
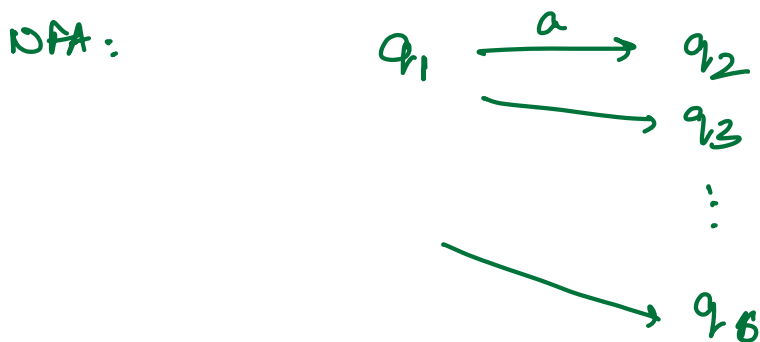
\downarrow C remains unreachable



NFA \leftarrow
 Non Deterministic Finite Automata

$L_1 \rightarrow \text{DFA}$
 $(\text{DFA})^R \rightarrow L_1^R \rightarrow \underline{\text{DFA or NFA}}$

Non Deterministic Finite Automata:



DFA: $\delta: Q \times \Sigma \rightarrow Q$

NFA: $\delta: Q \times \Sigma \rightarrow 2^Q$

$Q: \{A, B\}$

$\Sigma: \{a, b\}$

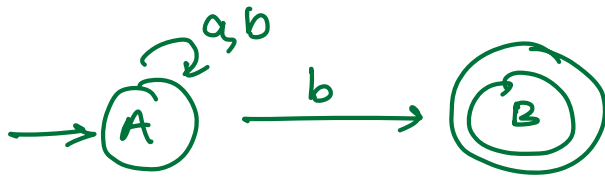
$Q \times \Sigma$	2^Q
-------------------	-------

Subset	
$\{1, 2, 3\}$	2^3
$\{1\}$	$\{12\}$
$\{2\}$	$\{13\}$
$\{3\}$	$\{23\}$
$\{1, 2\}$	$\{123\}$

(A, a)
 (A, b)
 (B, a)
 (B, b)

$\{\emptyset\}$ → no transition
 $\{A\}$ goes to A
 $\{B\}$ goes to B
 $\{AB\}$ goes to both AB

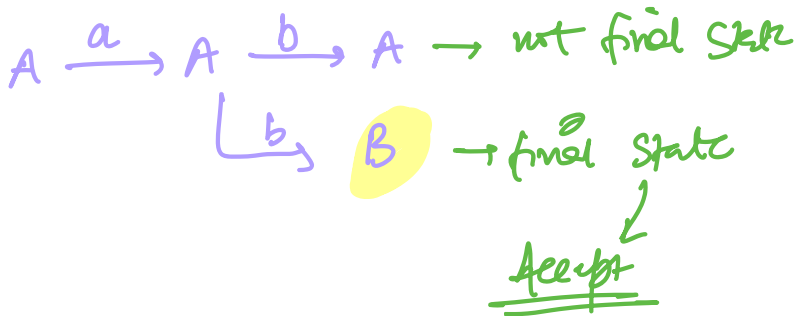
Eg: NFA $\Sigma = \{a, b\}$
 strings wds with 'b'



$Q \times \Sigma$	
(A, a)	A
(A, b)	AB
(B, a)	\emptyset
(B, b)	\emptyset

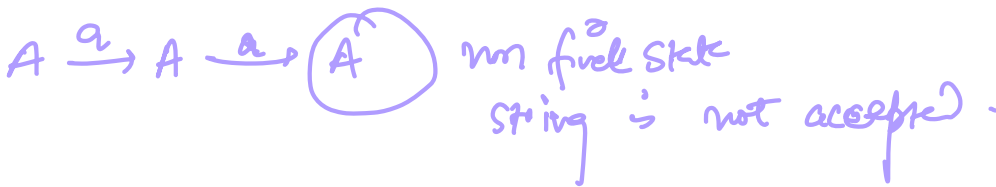
String Acceptance

ab

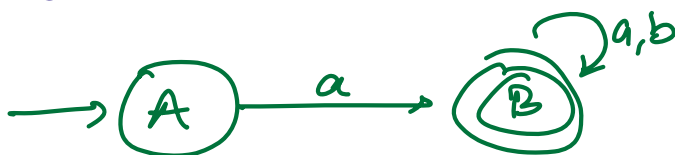


$\delta(B, a) = \emptyset$

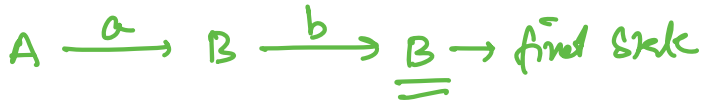
aa



Eg: Starts with a

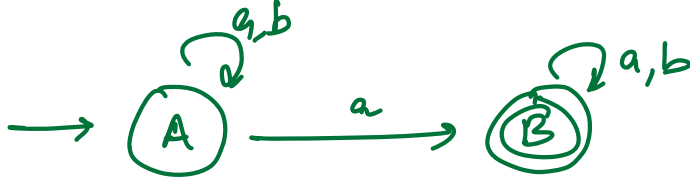


ab

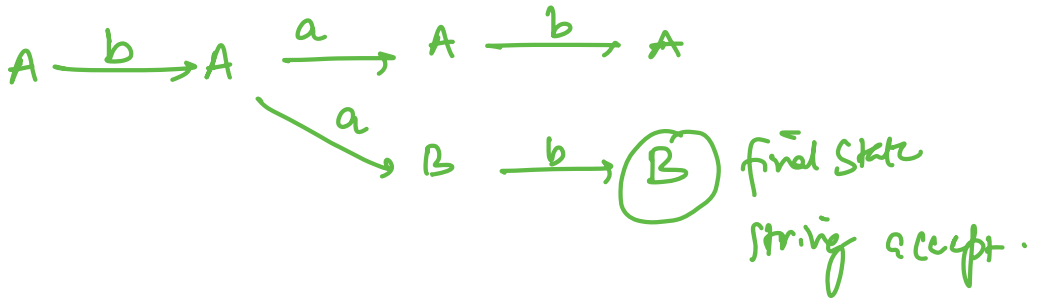


Eg:

contains a

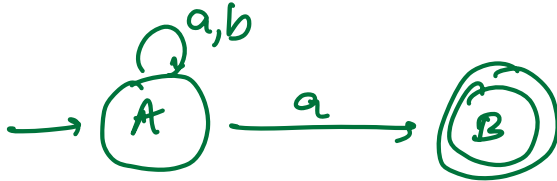


bab



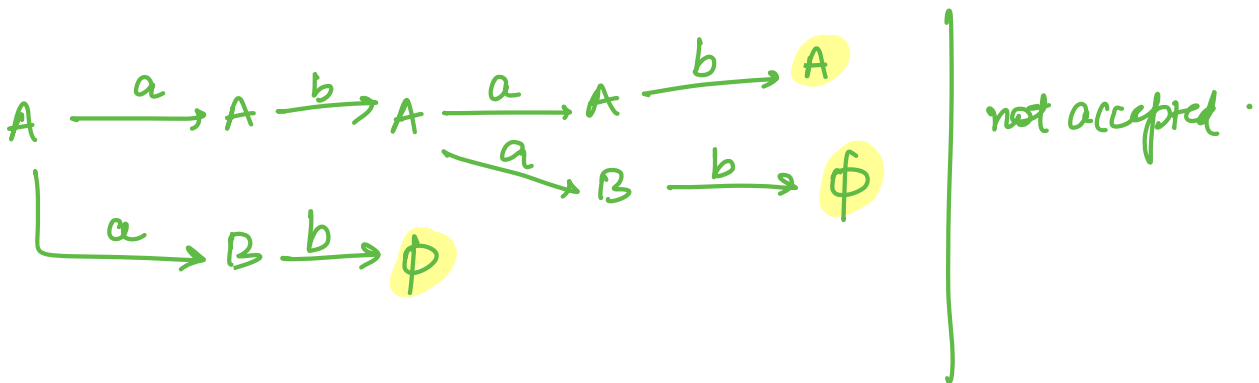
Eg:

ends with a



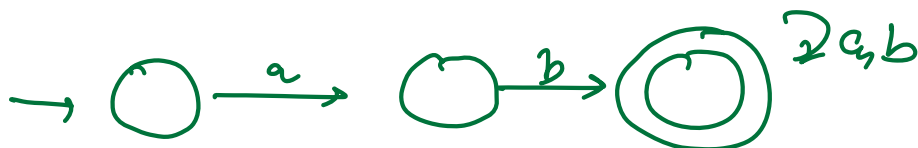
ababbbba

abab



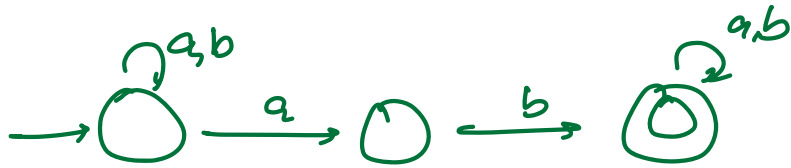
Eg:

starts with 'ab' $\Sigma = \{a, b\}$



Ex:

contains 'ab' $\Sigma = \{a, b\}$



Ex:

ends 'ab' $\Sigma = \{a, b\}$

